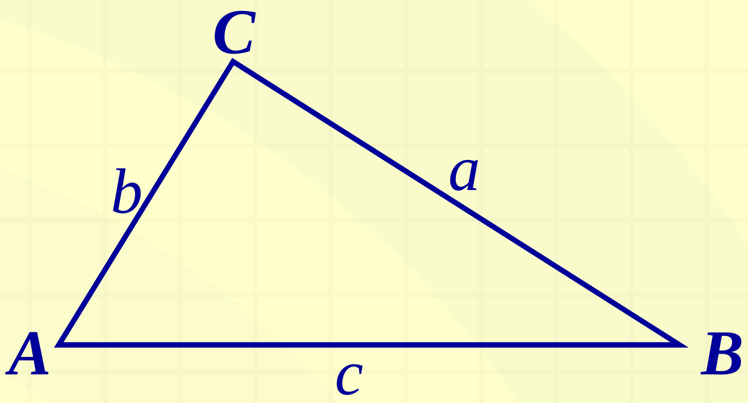
Digital Lesson

Law of Cosines

Instructions

- Please go through the digital lesson with worked out examples
- Complete the given assignment at the end of the lesson

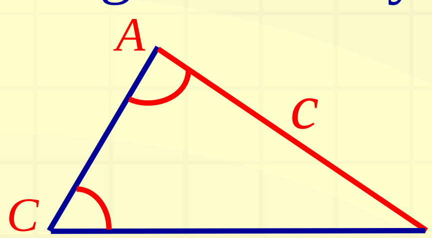
An **oblique triangle** is a triangle that has no right angles.

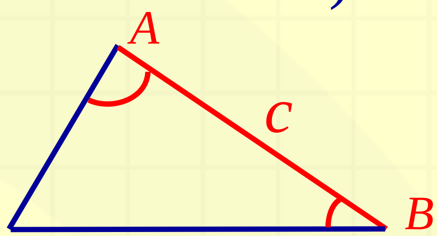


To solve an oblique triangle, you need to know the measure of at least one side and the measures of any other two parts of the triangle – two sides, two angles, or one angle and one side.

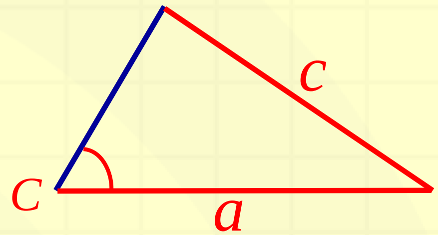
The following cases are considered when solving oblique triangles.

1. Two angles and any side (AAS or ASA)

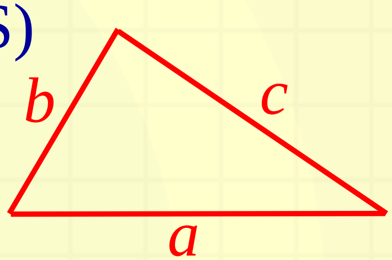




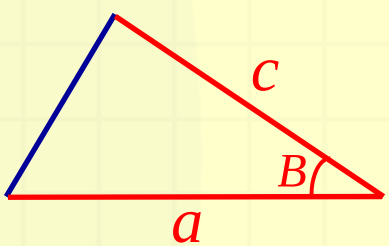
2. Two sides and an angle opposite one of them (SSA)



3. Three sides (SSS)



4. Two sides and their included angle (SAS)



The last two cases (SSS and SAS) can be solved using the **Law of Cosines**.

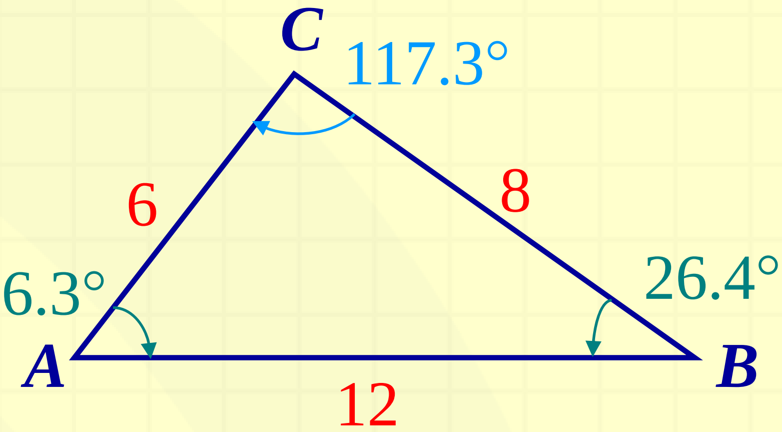
(The first two cases can be solved using the Law of Sines.)

Law of Cosines

Standard Form		Alternative Form
$a^2 = b^2 + c^2 - 2bc\cos A$	→	$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$
$b^2 = a^2 + c^2 - 2ac\cos B$	→	$\cos B = \dfrac{a^2 + c^2 - b^2}{2ac}$
$c^2 = a^2 + b^2 - 2ab\cos C$	→	$\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

Example:

Find the three angles of the triangle.



Find the angle opposite the longest side first.

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$= \frac{8^2 + 6^2 - 12^2}{2(8)(6)}$$

$$= \frac{64 + 36 - 144}{96}$$

$$= \frac{-44}{96}$$

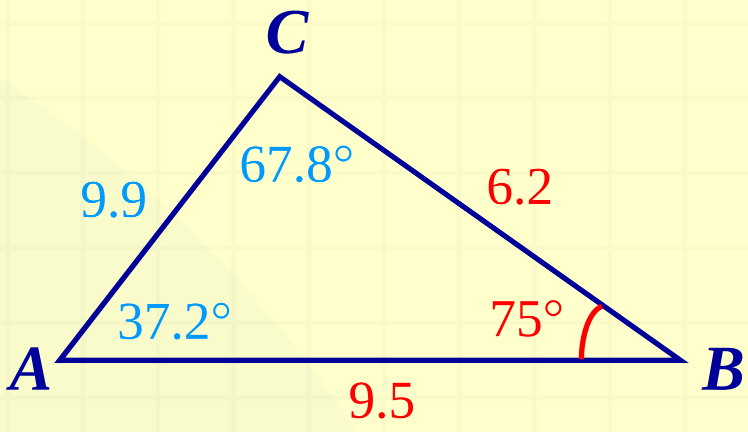
$$C \approx 117.3°$$

Law of Sines: $\frac{12}{\sin 117.3°} = \frac{6}{\sin B}$ ⟶ $B \approx 26.4°$

$$A \approx 180° - 117.3° - 26.4° = 36.3°$$

Example:

Solve the triangle.



Law of Cosines:

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$= (6.2)^2 + (9.5)^2 - 2(6.2)(9.5)\cos 75°$$

$$\approx 38.44 + 90.25 - (117.8)(0.25882)$$

$$\approx 98.20$$

$$b \approx 9.9$$

Law of Sines: $\dfrac{9.9}{\sin 75°} = \dfrac{6.2}{\sin A} \longrightarrow A \approx 37.2°$

$$C \approx 180° - 75° - 37.2° = 67.8°$$

Heron's Area Formula

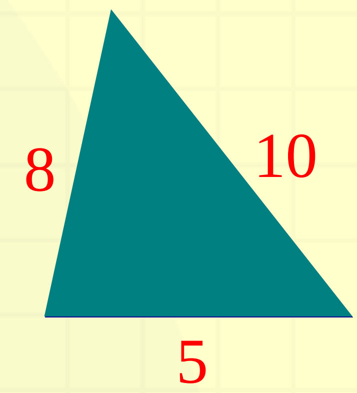
Given any triangle with sides of lengths *a*, *b*, and *c*, the area of the triangle is given by

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

where $s = \dfrac{a+b+c}{2}$.

Example:

Find the area of the triangle.



$$s = \frac{a+b+c}{2} = \frac{5+8+10}{2} = 11.5$$

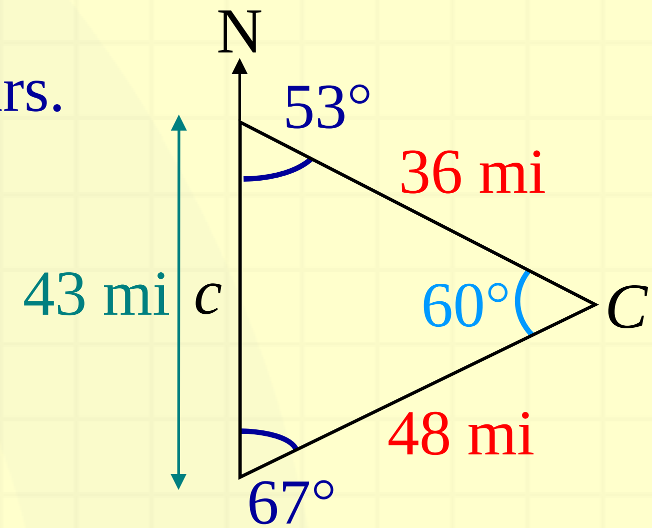
$$\text{Area} = \sqrt{11.5(11.5-5)(11.5-8)(11.5-10)}$$

$$= 19.8 \text{ square units}$$

Application:

Two ships leave a port at 9 A.M. One travels at a bearing of N 53° W at 12 mph, and the other travels at a bearing of S 67° W at 16 mph. How far apart will the ships be at noon?

At noon, the ships have traveled for 3 hours.



Angle C = 180° – 53° – 67° = 60°

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$= 36^2 + 48^2 - 2(36)(48)\cos 60° = 1872$$

$$c \approx 43 \text{ mi}$$

The ships will be approximately 43 miles apart.

Assignment

- Finish the problems given on the following slides. Thanks!

Vocabulary Check

Fill in the blanks.

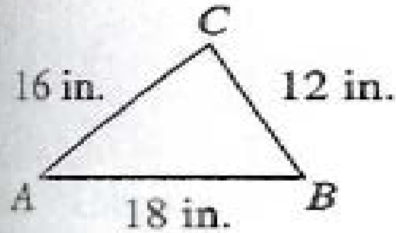
1. The standard form of the Law of Cosines for $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$ is ______ .

2. ______ Formula is established by using the Law of Cosines.

3. Three different formulas for the area of a triangle are given by Area = ______ , Area $= \frac{1}{2}bc \sin A = \frac{1}{2}ab \sin C = \frac{1}{2}ac \sin B$, and Area = ______ .

In Exercises 1–20, use the Law of Cosines to solve the triangle.

1.



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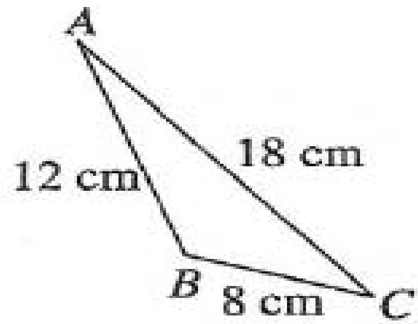

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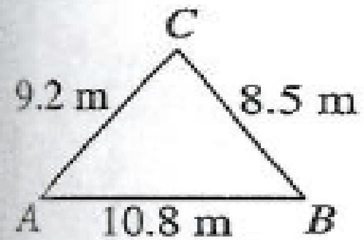

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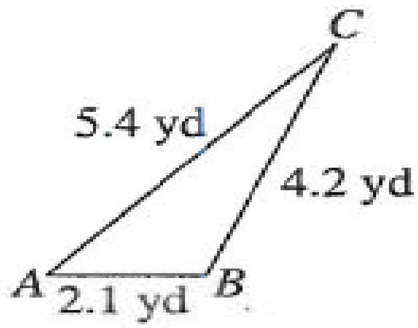

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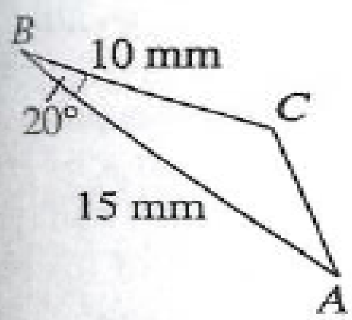

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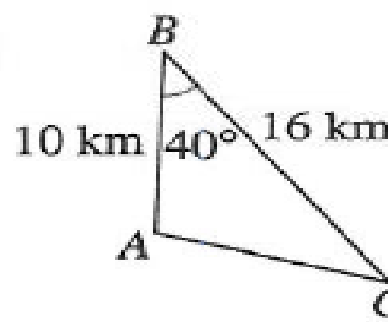


9. $a = 6, \quad b = 8, \quad c = 12$

10. $a = 9, \quad b = 3, \quad c = 11$

11. $A = 50°, \quad b = 15, \quad c = 30$

12. $C = 108°, \quad a = 10, \quad b = 7$

13. $a = 9, \quad b = 12, \quad c = 15$

14. $a = 45, \quad b = 30, \quad c = 72$

15. $a = 75.4, \quad b = 48, \quad c = 48$

16. $a = 1.42, \quad b = 0.75, \quad c = 1.25$

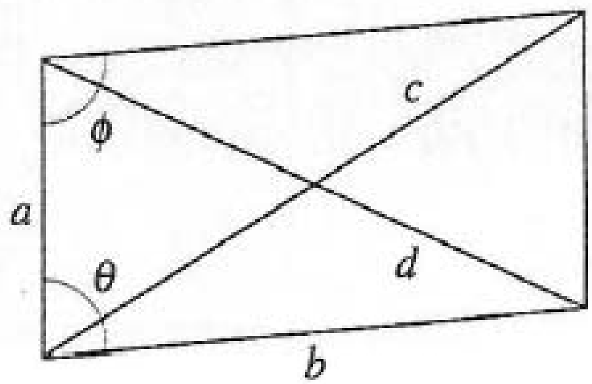
17. $B = 8° 15', \quad a = 26, \quad c = 18$

18. $B = 10° 35', \quad a = 40, \quad c = 30$

19. $B = 75° 20', \quad a = 6.2, \quad c = 9.5$

20. $C = 15° 15', \quad a = 6.25, \quad b = 2.15$

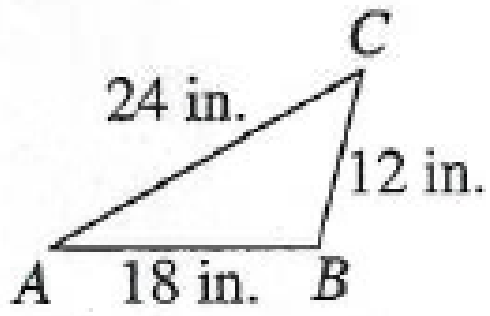
In Exercises 21–26, complete the table by solving the parallelogram shown in the figure. (The lengths of the diagonals are given by c and d.)



	a	b	c	d	θ	ϕ
21.	4	8	[illegible]	[illegible]	30°	[illegible]
22.	25	35	[illegible]	[illegible]	[illegible]	120°
23.	10	14	20	[illegible]	[illegible]	[illegible]
24.	40	60	[illegible]	80	[illegible]	[illegible]
25.	15	[illegible]	25	20	[illegible]	[illegible]
26.	[illegible]	25	50	35	[illegible]	[illegible]

In Exercises 27–36, use Heron's Area Formula to find the area of the triangle.

27.



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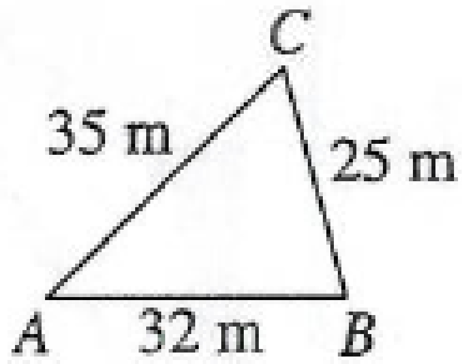
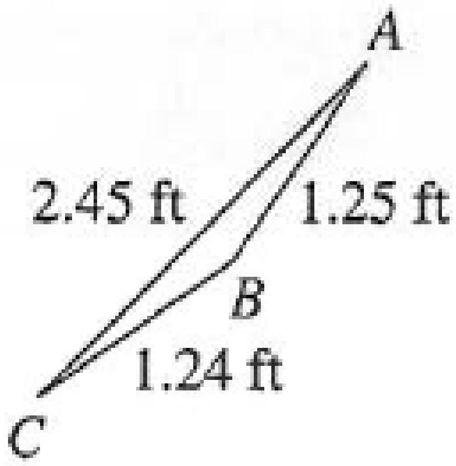


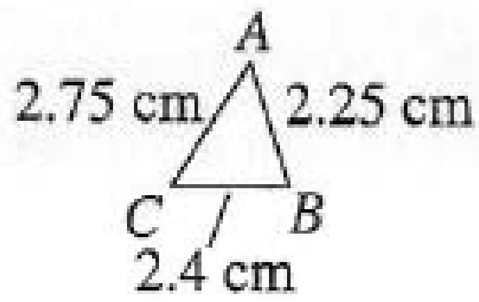
29. $a = 5, \quad b = 8, \quad c = 10$

30. $a = 14, \quad b = 17, \quad c = 7$

31.



32.



33. $a = 3.5, \quad b = 10.2, \quad c = 9$

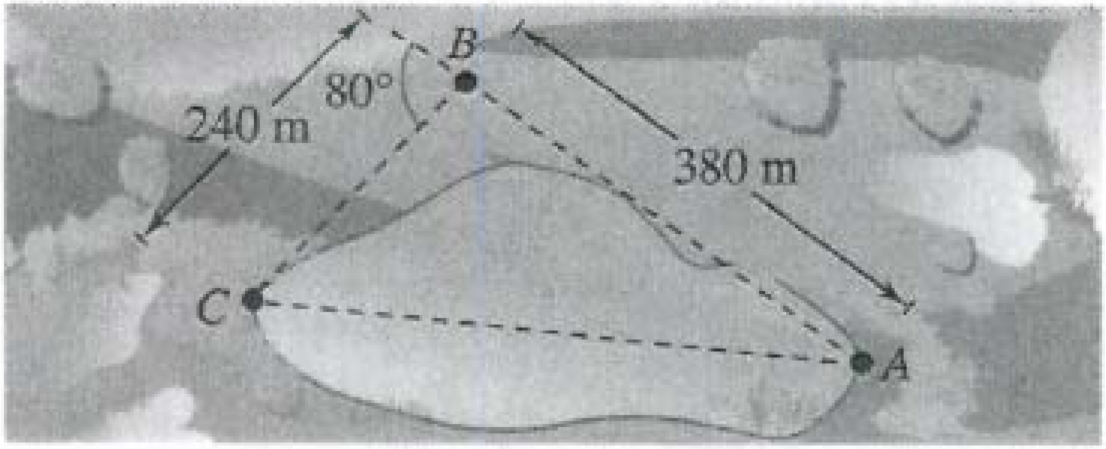
34. $a = 75.4, \quad b = 52, \quad c = 52$

35. $a = 10.59, \quad b = 6.65, \quad c = 12.31$

36. $a = 4.45, \quad b = 1.85, \quad c = 3.00$

37. ***Navigation*** A plane flies 810 miles from Franklin to Centerville with a bearing of 75° (clockwise from north). Then it flies 648 miles from Centerville to Rosemont with a bearing of 32°. Draw a diagram that visually represents the problem, and find the straight-line distance and bearing from Rosemont to Franklin.

38. ***Surveying*** To approximate the length of a marsh, a surveyor walks 380 meters from point A to point B. Then the surveyor turns 80° and walks 240 meters to point C (see figure). Approximate the length AC of the marsh.



39. ***Navigation*** A boat race runs along a triangular course marked by buoys A, B, and C. The race starts with the boats headed west for 3600 meters. The other two sides of the course lie to the north of the first side, and their lengths are 1500 meters and 2800 meters. Draw a diagram that visually represents the problem, and find the bearings for the last two legs of the race.

40. ***Streetlight Design*** Determine the angle θ in the design of the streetlight shown in the figure.